Normalized PSD Difference Method

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WWW.ESPEC.COM
303.254.8800

Normalized PSD Difference Method

by Matt Giaraffa
Former Senior Mechanical Engineer
Formerly Qualmark Corporation | Now Product and Brand of ESPEC North America

ABSTRACT

In HALT & HASS testing (Highly Accelerated Life Test & Highly Accelerated Stress Screen), the PSD (Power Spectral Density) of input energy into a tested component is important, as it measures which frequencies of the product under test have become stimulated. Coinciding, there is a need to compare the similarity of two PSD curves, yet no commonly accepted metric exists for comparing the similarity or dissimilarity of two PSD curves. Fulfilling this void, a new method of PSD comparisons was created at Qualmark, now making it possible to answer the question of "how different are these PSD's?". This new method normalizes two samples, plots the difference, and shows energy shifts across the frequency spectrum. With an improved metric to effectively compare these differences, deeper analysis can be performed to move PSD comparisons from subjective to objective. This method will assist HALT & HASS users to monitor PSD changes over time, develop PSD tolerances, evaluate variability between products in HASS, and also provide an effective tool to assess table performance.

INTRODUCTION

While performing product reliability testing, either during prototyping with HALT, or during production with HASS, the goal is to apply vibration energy that excites natural frequencies of the product. Exciting natural frequencies of the product causes components to vibrate, and in turn will accelerate mechanical fatigue failures. In order to catalyze resonance of the product's natural frequencies, the PSD of the input energy is important- there is limited need to put a lot of energy into vibrating frequencies that do not cause resonance in the product. As the frequency content of the input energy is important, it can be useful to compare two PSD's, whether watching for changes over time, comparing new components, etc. Current methods of frequency response comparison are not effective tools to show the differences between two PSD's. The new Normalized PSD Difference method is a concise, easy to see way to show energy shifts across the frequency band between two measured response curves.

PSD DIFFERENCE

The purpose of the Normalized PSD Difference is to show where and to what magnitude vibration energy shifts across frequencies between two responses.

CALCULATION METHOD - Creating a Normalized PSD Difference is calculated by

- Calculating the power in g^2RMS of two PSD curves
- Normalizing the two curves to the same power level
- Series one values are subtracted from series two values and plotted as a curve
- The area under the difference curve calculated as a metric of total amount of difference

<u>Power</u> - Simplified, calculating the power is summing the area under the curves, and in detail, the method is shown below in Equation 1 and Figure 1 using a trapezoid integration method, derived from NASA FEMCI.

$$dB = 10\log\left(PSD_H / PSD_L\right)$$

$$\#Octaves = \frac{\log\left(F_H / F_L\right)}{\log(2)}$$

$$m = dB / \#Octaves$$

if $m \neq -10\log(2)$:

$$P = \int 10\log(2)\,\frac{PSD_H}{10\log(2) + m}\left[F_H - F_L\left(\frac{F_H}{F_L}\right)^{m / 10\log(2)}\right]dF$$

if $m = -10\log(2)$:

$$P = \int D_L \times F_L \times \log\left(\frac{F_H}{F_L}\right)dF$$

EQUATION 1: Calulating PSD Curve Area

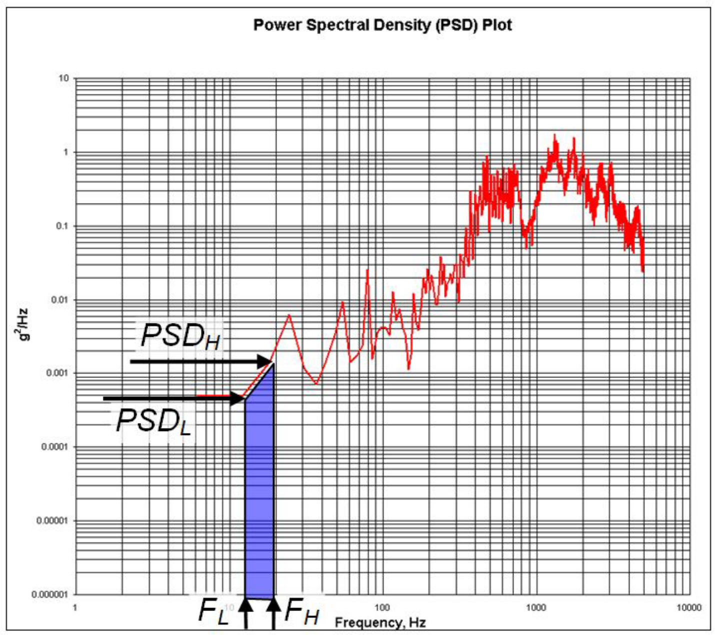

FIGURE 1: Power Integration Variables

P is the area under the PSD curve, also the total power in units of g^2RMS. This unit is used for calculating PSD Difference discussed below. However, to convert to the more popular gRMS units, simply take the square root.

<u>Normalized Power</u> - Normalizing both curves scales each to 1 gRMS. Once scaled, the difference between each curve is purely shifting power across the frequency band, and not a false difference due to differing gRMS levels. If this was not done, the difference curve would show large areas of difference that were due to power level and not frequency shifts. Figure 2 shows a typical PSD curve in red, and the normalized version of the same response curve in black.

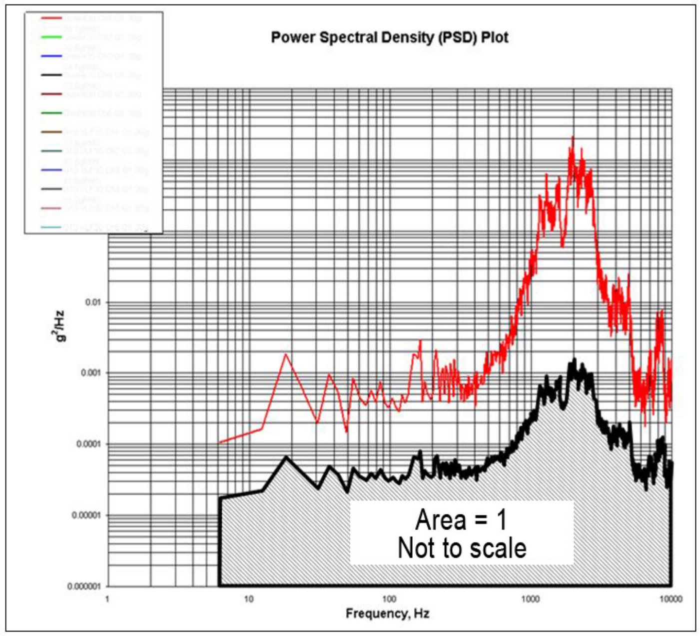

FIGURE 2: Normalize Curve to 1 (unit less)

To normalize the curve, follow Equation 2:

$$\begin{aligned}
&\textit{for } i = 1 \textit{ to } n \\
&\textit{where } n = \textit{number of data points} \\
&P_{Ni} = P_{Oi} / P_T \\
&P_{Ni} = \textit{Normalized power value of } i \ (1/Hz) \\
&P_{Oi} = \textit{Non-normalized power value of } i \ (g^2RMS/Hz) \\
&P_T = \textit{Total power } (g^2RMS)
\end{aligned}$$

EQUATION 2: Normalized Power

If the power is normalized correctly, summing the area of the normalized curve using Equation 2 will give a result of 1.

Difference - For the second step, once the two curves are normalized to the same power level of 1, the difference is calculated. The method is simple, but has an important preliminary requirement: both data sets must have the same X values, i.e. both must be recorded with the same frequency limits and frequency resolution. The examples used in this report are recorded from 6 - 5005 Hz at 6.1 Hz resolution. If the X values are not the same between the two data sets, the calculated difference will be comparing two points that are not at the same frequency. After making sure that both data sets have the same X values, the difference is:

$$\begin{aligned}
&\textit{from } i = 1 \textit{ to } n \\
&\textit{where } n = \textit{number of data points} \\
&\qquad P_{N2i} - P_{N1i} \\
&P_{N1i} = \textit{Series 1 Normalized Power value of } i \ (1/Hz) \\
&P_{N2i} = \textit{Series 2 Normalized Power value of } i \ (1/Hz)
\end{aligned}$$

EQUATION 3: Difference

Normalized Total Difference - In order to produce one number that answers the question "how different are these?", Normalized Total Difference sums the area under the difference curve and divides it by the frequency band measured. Without normalizing, the total difference number between the same two PSD's would change depending on what the bandwidth measured is. Area of difference is calculated using the trapezoid method, below in Equation 4. Definitions of variables are shown in Figure 3.

$$dB = 10 \log (D_H / D_L)$$

$$\#Octaves = \frac{\log (F_H / F_L)}{\log (2)}$$

$$m = dB / \#Octaves$$

If $m \neq 10 \log (2)$:

$$D = \int \left| 10 \log (2) \frac{DH}{10 \log (2) + m} \left[F_H - F_L \left(\frac{F_H}{F_L} \right)^{m / 10 \log (2)} \right] \right| dF$$

If $m = -10 \log (2)$:

$$D = \int \left| D_L \; x \; F_L \; x \; \log \left(\frac{F_H}{F_L} \right) \right| dF$$

EQUATION 4: Total Difference

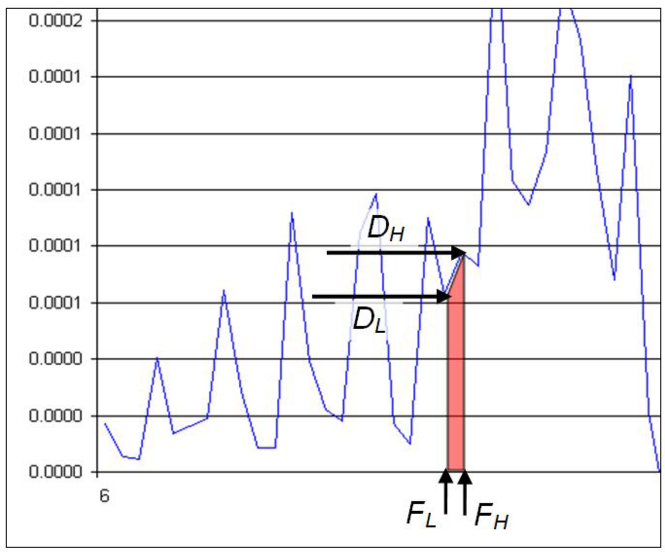

FIGURE 3 Difference Area Variables

D is the Total Difference (unit less), and D divided by frequency bandwidth is Normalized Total Difference (1/Hz).

$$D_N = D / BW$$

D_N = *Normalized Total Difference (1/Hz)*

D = *Total Difference (unit less)*

BW = *Frequency Bandwidth (Hz)*

EQUATION 5: Normalized Total Difference

EXAMPLE - Starting out by normalizing a power curve, following Equation 2, the blue line is reduced to an area of 1 in Figure 4. Channel 1 has an area under the PSD curve of 3082 g^2RMS, which is the P_T value of Equation 2, in this case.

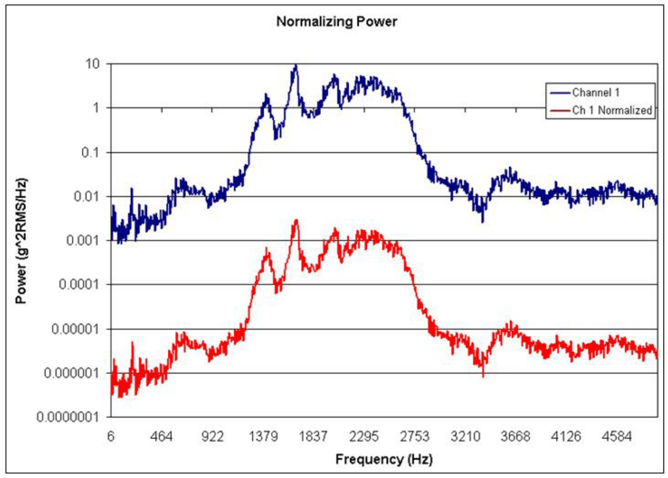

FIGURE 4: Example Normalized Power

Once a second response plot is normalized, the difference is calculated using Equation 3. The function created by Equation 3 is plotted and shown in Figure 5.

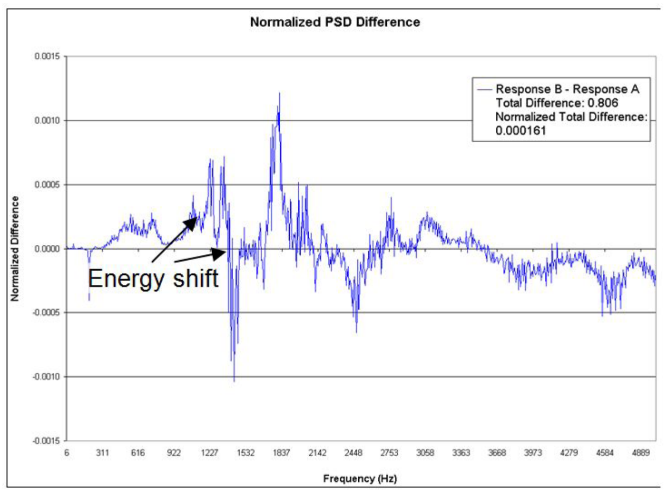

FIGURE 5: Example Difference Curve

Looking at the difference curve in Figure 5, it can be seen that between the two curves compared, most of the difference is small and spread out. Where the difference curve is greater than zero, Response B has more energy, and where the difference curve is less than zero, Response A has more energy. There are a few shifts of energy worth noting: from about 1000 Hz to 1500 Hz, there is a larger response that exists at a higher frequency in Response A than B; there is also a cluster of power shifted from 1700 Hz in B up to 2400 Hz in A.

Also shown above in Figure 5 are the Total Difference and Normalized Total Difference. The former is unit less sum of the area under the difference curve, giving an indicator of the magnitude of total energy shifts. And the latter takes the Total Difference and divides by the frequency bandwidth, following Equation 5.

<u>ACCEPTABLE DIFFERENCE</u> - Now that there exists a method to easily see differences in response, the next question is: “what is acceptable?”. In this case, it is not as simple as above where there are a couple equations that are solved to produce an answer. The most important point to consider is whether the natural frequencies of the products

under test are being excited. Following this, the most important frequencies in the difference plot are where the products resonances are excited. While the acceptable change is a function of the application, look for peaks and clusters of energy shifting across the frequency band. An example is shown below in Figure 6 with the X, Y and Z direction accelerations plotted.

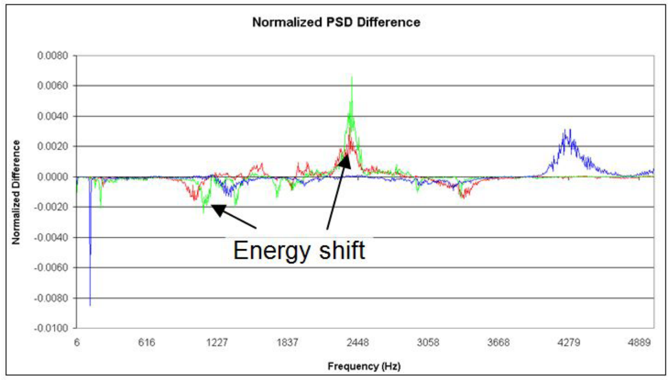

FIGURE 6: Frequency Shift Example

As seen in Figure 6, there are several shifts between the two responses. If the product tested had several important natural frequencies in the 2400 Hz range, the change depicted may be an issue.

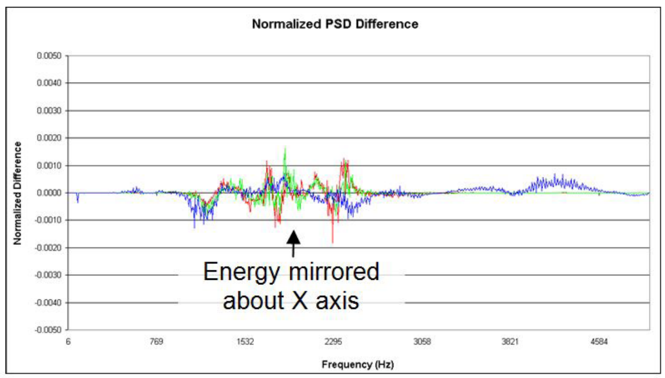

FIGURE 7: Reflected Energy Example

A different situation is shown in Figure 7, where most of the difference is in the form of energy "reflected" about the X axis. Instead of shifting power to different frequency ranges all together, in this example, most of the change is the energy switching axes, but staying in the same frequency band. This is most likely not an issue that changes the resonances that become excited.

CONCLUSION

With the new Normalized PSD Difference method, a standard way of comparing where energy shifts between two PSD's curves exist. Comparisons can be made when looking at frequency response curves. From comparing the response of multiple products in a HASS fixture to watching for change in PSD of a bare HALT/HASS table over time, the Normalized PSD Difference method quickly shows where frequency shifts exist, and to what magnitude.

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REFERENCES

NASA FEMCI Book. 10 May 2009. NASA Finite Element Modeling Continuous Improvement. 12 May 2009 <http://femci.gsfc.nasa.gov/random/randomgrms.html>.

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